

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

TMA 1111 – MATHEMATICAL TECHNIQUES

(All sections / Groups)

7 OCTOBER 2016
9.00 A.M. – 11.00 A.M.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **5 pages** only including the cover page with **5 Questions** and an **Appendix**.
2. There are **2 Sections**:
Section A (3 Questions): Attempt **ALL THREE (3)** questions.
Section B (2 Questions): Attempt **ONE (1)** question **ONLY**.
All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write your answers in the Answer Booklet provided, and **start each question on a new page**.
4. Show all steps.
5. Formulas are provided and attached in Appendix.

SECTION A**Answer ALL THREE (3) Questions.****Question 1**

- a. Find the angle between the vectors $u = i - 2j + 2k$ and $v = -3i + 2j + k$. [2 marks]
- b. Consider the points $P = (-1, 2, 1)$, $Q = (0, -3, 2)$, and $R = (1, 1, -4)$.
- Find the vectors $u = \vec{PQ}$ and $v = \vec{PR}$. [3 marks]
 - Compute the cross product of \vec{PQ} and \vec{PR} . [2 marks]
- c. Let K be the line passing through $A(5,0,-2)$ that is parallel to the line joining $B(2,3,-1)$ and $C(-1,0,1)$. Determine the parametric equation of the line K . [3 marks]

Question 2

- a. The first order differential equation (ODE) is given below:

$$3 \frac{dy}{dx} = \frac{x^3}{y^2 - 1}$$

- Show that the ODE is separable. [1 mark]
 - Solve the ODE using separable method. [2 marks]
- b. Find the general solution of the following homogeneous linear second order differential equation.
- $$2y'' - 4y' + 2y = 0$$
- [3 marks]
- c. Solve the exact differential equation, $x^2 y^4 dx + \frac{4}{3} x^3 y^3 dy = 0$. [4 marks]

Question 3

A researcher would like to investigate the relationship between the daily smart phone usage duration (measured in hours), x , and the sleep quality (measured in a certain Quality Sleep Index), y , among university students. The summary of the sample data collected by this researcher is as follows:

$$n = 12, \sum x = 124, \sum y = 124, \sum x^2 = 1578, \sum y^2 = 1568, \sum xy = 1045$$

- Compute \bar{x}, \bar{y}, S_{xy} and S_{xx} . [4 marks]
- Find the regression model, $\hat{y} = \beta_0 + \beta_1 x$, to fit the data. [3 marks]
- What is the value of correlation coefficient, r ? What does this value indicate? [3 marks]

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SECTION B

Answer ONE (1) Question ONLY.

Question 4

a. Given $f(x) = \begin{cases} 6x - 1, & x < 2 \\ 4x + 3, & x \geq 2 \end{cases}$.

Compute $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$. Hence, determine whether $\lim_{x \rightarrow 2} f(x)$ exists by applying the suitable limit theorem(s) or rule(s). [3 marks]

b. By applying the appropriate technique(s) or rule(s), compute $\lim_{x \rightarrow \infty} \frac{5x + 1}{7x - 3}$. [2 marks]

c. By using the appropriate technique(s), find

i. the second derivative (y'') of $y = \frac{(x-1)(x^5 - 3x^2)}{x^2}$. [2 marks]

[Hint: Simplify the function first may be easier to find the derivative.]

ii. the derivative of $y = (2x^4 + e^x)^3$. [Hint: Chain rule may be useful.] [1 mark]

iii. $\frac{dy}{dx}$ for $xy + \ln x = 0$. [2 marks]

Question 5

a. Find $\int \frac{(3x^2 - 4)^2}{x^2} dx$ by using appropriate integration technique. [2 marks]

b. Use the *Fundamental Theorem of Calculus* to compute $\int_{-1}^0 (2x^3 - 2x + 3) dx$. [2 marks]

c. Use *integration by substitution* to compute $\int 3x^2 \sqrt{1 + x^3} dx$. [3 marks]

d. Use *integration by part* to compute $\int 2x^3 \cos x dx$. [3 marks]

Continued.....

APPENDIX

1. Length of a vector \mathbf{v} : $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$
2. $\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$
3. Dot product: $\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$
4. Parametric equation: $x = x_0 + at$; $y = y_0 + bt$; $z = z_0 + ct$
5. Criterion for an Exact Differential Equation: $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
6. General solution of the homogeneous linear second order differential equation
 For Distinct real roots: $y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$
 For Repeated roots: $y = c_1 e^{\lambda_1 x} + c_2 x e^{\lambda_1 x}$
 For Complex conjugate roots: $y = e^{\alpha x} (c_1 \cos \beta x + c_2 \sin \beta x)$.
7. Derivative Formulas

$\frac{d}{dx} e^x = e^x$
$\frac{d}{dx} \ln x = \frac{1}{x}$
Power Rule: $\frac{d}{dx} x^n = nx^{n-1}$ if $f(x) = x^n$ with $n \in \mathbb{R}$
Product Rule: $\frac{d}{dx} (f \cdot g) = f \frac{dg}{dx} + g \frac{df}{dx}$
Chain Rule: If $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Continued.....

8. Integration Formulas

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \text{ for } n \neq -1, n \text{ rational}$$

$$\int \frac{1}{x} dx = \ln x + C$$

$$\int e^x dx = e^x + C$$

$$\int \sin kx dx = -\frac{\cos kx}{k} + C$$

$$\int \cos kx dx = \frac{\sin kx}{k} + C$$

Integration by Substitution: If $u = g(x)$ and $du = g'(x)dx$, then

$$\int f(u)du = \int f(g(x)) \frac{d}{dx} g(x) dx$$

Integration by Part: $\int u dv = uv - \int v du$

$$9. \quad \bar{x} = \frac{\sum x}{n}$$

$$S_{xy} = \sum xy - \frac{(\sum x \sum y)}{n}, \quad S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n}, \quad S_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$10. \quad \beta_1 = \frac{S_{XY}}{S_{XX}}, \quad \beta_0 = \bar{y} - \beta_1 \bar{x}$$

$$11. \text{ Sample correlation coefficient } = r = \frac{S_{xy}}{\sqrt{S_{xx}} \sqrt{S_{yy}}}$$

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